Using Katamino and Pentominoes at KS2

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The author gratefully acknowledges the contribution made by Mark Humble of Brunel University

Introduction

Katamino is based on the 12 unique shapes 'pentominoes' that can be created by connecting 5 congruent squares. Pentominoes are said to have been 'invented' by Solomon W. Golomb in 1953 at a talk he gave for the Harvard Mathematics Club. They had in fact been around for a time. He brought them to the attention of a much wider audience and since then a vast number of problems have been set using these 12 intriguing shapes.

Their potential as a motivating teaching resource at Key Stage Two cannot be over-estimated and there follows a series of uses to develop children's mathematical concepts and thinking, their spatial awareness and their problem-solving strategies. Children of all abilities can use them. They certainly provide the level of challenge to inspire and motivate more able children.

In my experience as a classroom teacher I have often found that lower attaining mathematicians have a surprisingly high level of spatial ability. Operating at a similar level to more able children could do wonders for the weaker child's mathematical confidence.

Pentominoes can be used to develop children's understanding of the concepts of area and perimeter, transformational geometry including enlargement, congruence and symmetry, nets, volume and classification. All these will be illustrated through the various activities below. There are also some suggestions for the use of pentominoes in developing children's sense of number.

There is much ongoing research in education at the moment about developing children's thinking skills by approaching teaching and learning in a far more active and interconnected way. The development of thinking skills, such as classifying, hypothesising, reasoning, analysing etc., can all be encouraged through the use of these activities, which are all active and exploratory.

Pentomino problems provide children with a high level of challenge and encourage them to develop some of the necessary skills such as trial-and improvement strategies, perseverance, reasoning etc. To develop these skills in childhood would contribute immensely to our commitment to developing 'life-long learners' for this new millennium.

Through playing Katamino, children develop winning strategies based on their developing spatial awareness and learning to plan ahead in predicting the future moves of their opponents. There are many strategic games in use in schools and in the wider society and playing such games helps to develop children's strategic thinking- a key skill in many lines of work.

Starting Out with Pentominoes

The best thing about pentominoes is that there are only 12 of them, no more, no less. If we were to use tetrominoes (4 squares joined by the same rules) or hexominoes (6 joined squares), there would be only 5 or a confounding 35 different shapes respectively, one too few for much investigative work and one too great.

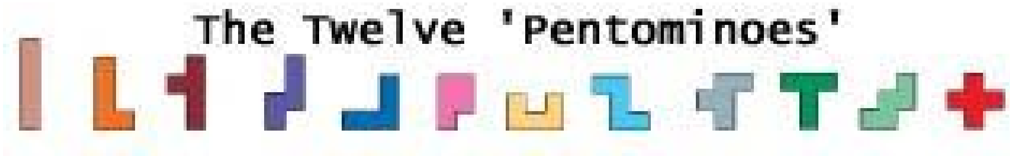
An interesting starting point for children is to see how many they can find after sharing with them what a pentomino is. I would not tell them how many there are at this point but would leave it open-ended to see what they come up with. Providing the children with squared paper and scissors for this activity means that they can draw them easily and then cut them out to ensure that they have not just found a rotation or reflection of another. This investigation requires children to be systematic in their approach. This could be demonstrated by finding all the tetrominoes together in the introductory activity. It is best tackled in pairs so that children can discuss the shapes and begin to classify them as they search for more.

Possible learning objectives for this lesson would include:

- *Solve mathematical problems by working in a systematic and co-operative way*
- *Recognise the congruence of shapes in different orientations*
- *Make and describe shapes and patterns*

Classifying pentominoes

The best way I have found for describing and identifying each shape is to liken it to a letter of the alphabet. This helps us to picture the various pentominoes and to classify them.



I L Y N V P U Z F T W X

We could now look at sorting our shapes by different criteria:

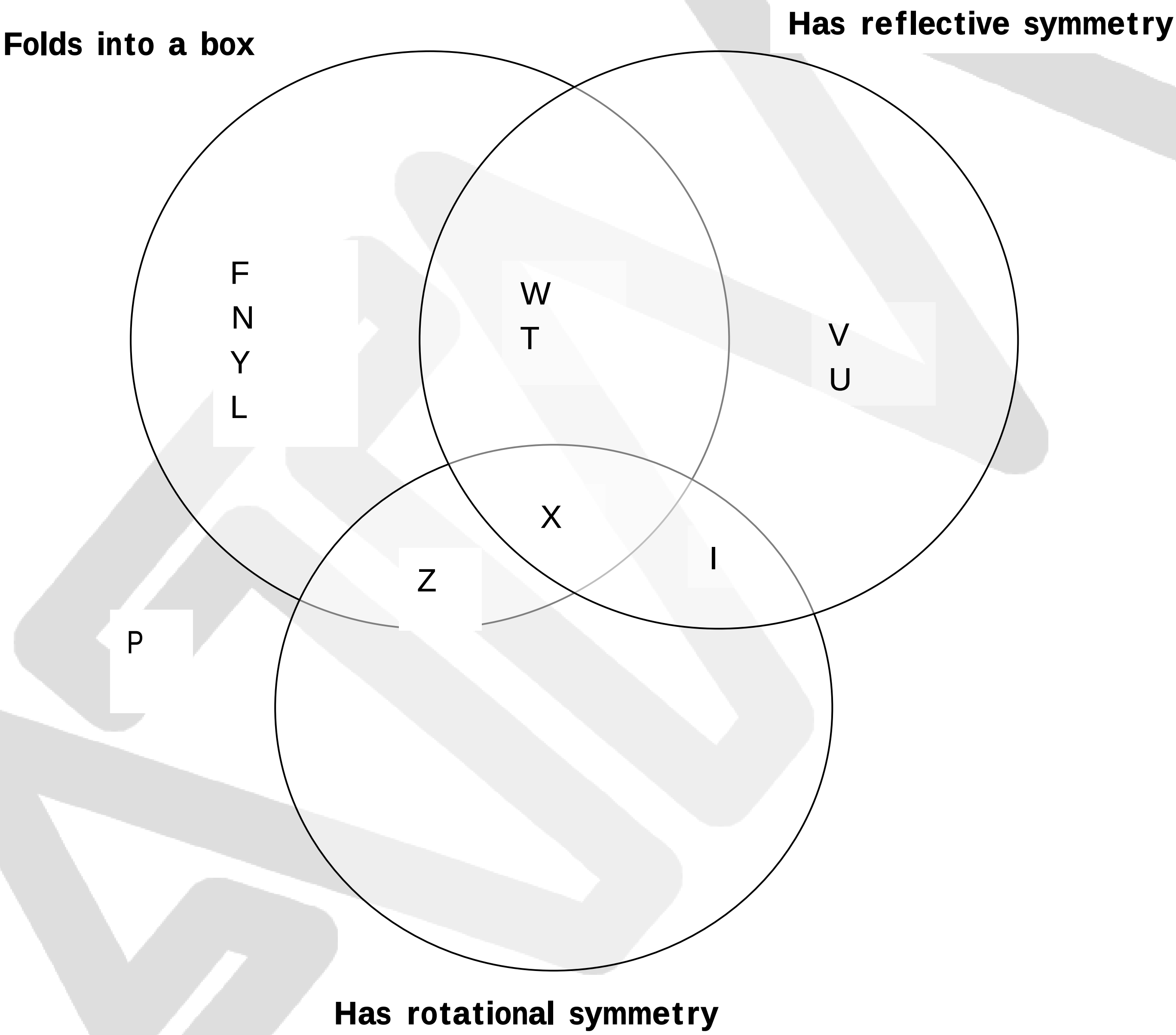
- *Length of perimeter (P is the only one that is 10, all the rest are 12)*
- *Whether or not they could be folded into an open top box*
- *Number of lines of reflective symmetry*
- *Order of rotational symmetry*
- *Number of sides- naming of irregular polygons*

It could be possible to then sort these 12 shapes by using a sorting method such as Venn Diagrams, Carroll Diagrams or a Tree Diagram.
e.g.

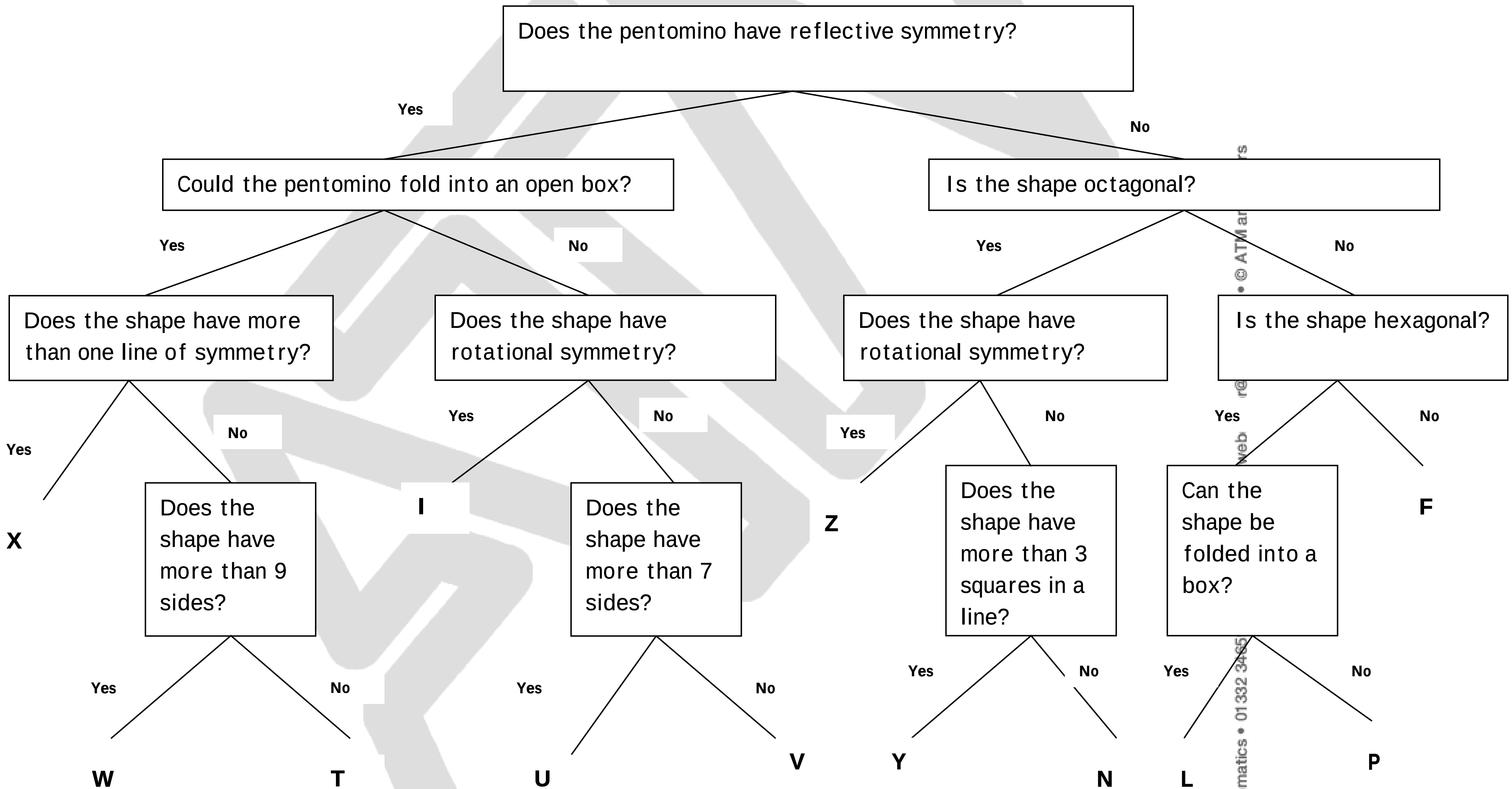
Carroll Diagram

	Folds into an open box	*Does not fold into an open box*
Has reflective symmetry	*T* *W* *X*	*I* *U* *V*
Does not have reflective symmetry	*F* *L* *N* *Z* *Y*	*P*

Venn Diagram



Tree Diagram



Activities along these lines could be used to develop the following learning objectives:

- *Classify and describe 2-D shapes referring to their properties*
- *Visualise 3-D shapes from 2-D drawings and identify different nets for an open cube*
- *Measure and calculate the perimeters of simple shapes*
- *Identify lines of symmetry in simple shapes and recognise shapes with no lines of symmetry*
- *Recognise rotational symmetry in simple shapes*

Using Katamino in the Classroom

Having now got the children familiar with some of the properties of pentominoes, it would be worth introducing the game and its related challenges at this point.

The advantage of Katamino's puzzleboard of 65 small squares is a divider to mark off different numbers of squares, providing a set of puzzles of varying difficulty. Thus placing the divider at 4/5 marks off a rectangle for 4 pentominoes to fit into. When a pupil has solved this, he or she moves the divider to 5/6 and fills the new rectangle with 5 pentominoes. And so on. Boards can therefore be adjusted to offer suitable challenges to pupils of widely differing spatial ability.

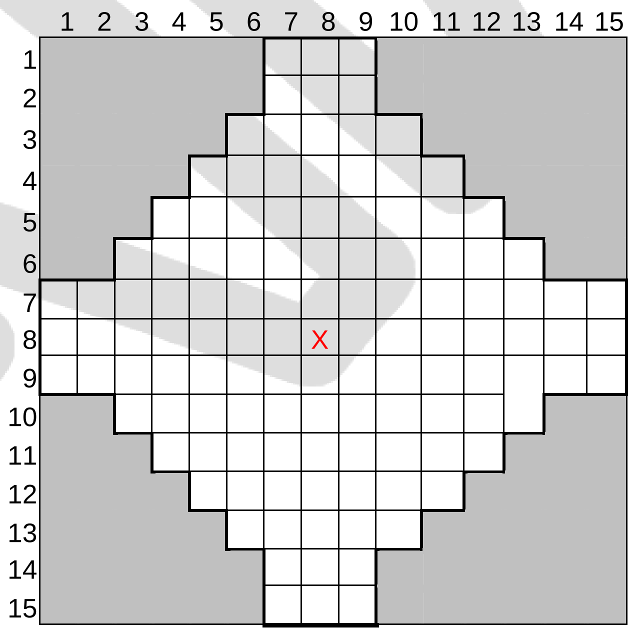
The grey board that contains 96 separate challenges to fit given pentomino pieces into different sized rectangular grids is naturally differentiated for all, depending on the size of the rectangle you choose to fill. Reasoning is developed as these challenges are attempted, e.g. you cannot leave a gap that is less than 5 squares on the board, certain shapes need fitting in earlier as they are more 'tricky' etc. The motivation to succeed grows with each successive success and the level of challenge increases. In fact it can become quite addictive!

Through attempting these challenges, teachers can draw pupils' attention to the links to area, and areas of rectangles, and the multiples of 5. It will also be sharpening pupils' spatial awareness, mathematical thinking and their problem-solving attitudes.

The Katamino game on the chequerboard shares out a set of pentominoes between two players. Each in turn plays a piece to *make it as difficult as possible for the opponent to find a place to play theirs.*

In playing the two-person game of Katamino, children will develop some of the skills of strategic game play and further develop their spatial awareness. In playing the game just several times with a friend, I realised that my strategies were becoming more sophisticated with each game and that I was better able to predict more moves ahead for both my opponent and myself. I began trying to block her moves and leave myself 'safe' moves by creating spaces that fitted only my shapes. I was also keen to use my perceived 'difficult' shapes first. This game has great potential for use in the classroom and would well be one the children would ask to borrow in wet breaks!

A further game, MasterKatamino© can be played using 2 sets of Katamino pentominoes and a special game board:



The game follows the same rules as Katamino, with players laying pentominoes alternatively, but the design of the board adds further challenge.

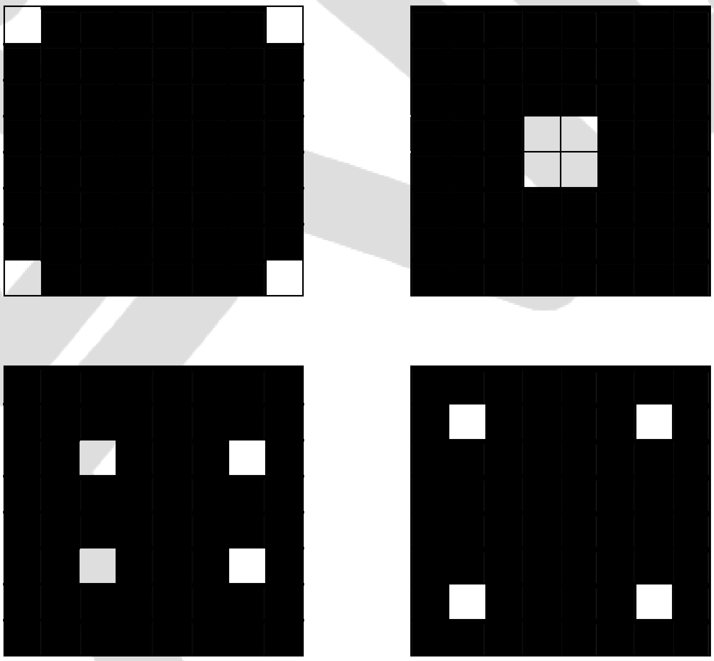
Katamino Challenges

Missing Pieces

Using the grey Katamino puzzle board, which gives you a ready made 5 x 13 rectangle, fit all the pieces in, leaving a 5 square gap somewhere in it which itself is in the shape of one of the pentomino pieces. In other words, the 5 empty squares are together on the board and create a pentomino shape. All the pentomino shapes can be 'created'.

Checkerboard Challenge

Using the 8 x 8 square in the Katamino set, fit all the pieces in. It is possible to do this in many ways so that the gaps you leave form symmetrical patterns, square tetrominoes etc. This square tetromino can be positioned anywhere in the checkerboard and a solution is still possible!



Pentominoes and Area

On the 65 squares of the puzzleboard pupils fill the 32 squares round the border with pentominoes. How many squares are left uncovered in the middle? Do different pentominoes round the edges leave different numbers of empty squares? Make a chart

Total squares	No of pentominoes	Unfilled squares
65	10	10
65	9	15 etc

Then put the divider in the end slot and repeat the experiment with the smaller area

60	?	?

Repeat, moving the divider along. What patterns are apparent? Then, leaving the puzzleboard, extend the exercise to squared paper

By using all 12 pentominoes, and ensuring that they touch by at least one side, what is the largest area you could enclose? For this activity, the children would need squared paper the same size as the pentomino squares. You could also include a grading system to encourage them to seek larger areas. e.g. Areas over 115 could score an A grade, areas from 105-114 could score B etc.

Pentomino Challenges

Enlarging Pentominoes

All the pentominoes, except V and X, can be enlarged by a factor of 2 by using 4 of the other pentomino pieces. The piece you are enlarging will not need to be used in this.
Each pentomino can be enlarged by a factor of 3 by using 9 of the other pentomino pieces. The piece you are enlarging will not need to be used in this.

This links in clearly to work on enlargements, and can be used to study the link between area and length when enlarging. (When doubling lengths, area quadruples etc.)

Congruent Pentominoes

It is possible to fit pairs of pentominoes together in such a way that they form the same shape as another pair of pentominoes. Encourage the children to find as many pairs as they can.

Indeed it is even possible to use all 12 pentominoes to create three identical and congruent shapes using 4 pentominoes for each- a real challenge! You could make it easier for them by giving them the shape!

Pentomino Rectangles

A link can be made to factors of 60 through challenging the children to make different rectangles using all 12 pentominoes. They could attempt to make rectangles with sides of 6 x 10, 5 x 12, 4 x 15 and 3 x 20. The last one is the most tricky as there is only three solutions to this.

Perimeters and Pentominoes

By combining two pentominoes by at least one side, can you make a shape that has a perimeter of 15? 19? More than 19?

Using any two pentomino pieces, challenge the children to combine them in such a way as to create the largest and the smallest possible perimeter. Using the same rules (any two pentominoes), can you find shapes with perimeters for all the lengths in between your lowest and highest perimeters?

3-D Pentomino Challenges

You can create (using Multilink?) or buy a three-dimensional set of pentominoes that can then be manipulated into various cuboids e.g. 3 x 4 x 5 or 2 x 5 x 6 or 2 x 3 x 10 cuboids.

The most able children could be asked to draw their set of pentominoes on isometric paper, a skill in itself!

All the pentominoes, except W and X can also be made into 3-D models using all 12 pentacubes. The models will be twice the width and length but three times the height. P is by far the easiest and has the greatest number of solutions so this would make a good starting point. L and N could then be attempted. F and T are much harder to solve! There is only 1 solution for F.

There are many other shapes that can be made using all 12 pentominoes, each of which have a volume of 60. Ideas for these can be found on many web sites.

Twenties

Can you cut up the following rectangle into the 12 pentominoes so that the total of numbers on each piece is 20? (Easily adaptable to other numbers and concepts)

2	3	7	5	8	8	1	6	1	7	5	9
1	5	1	4	1	2	2	6	2	3	4	5
5	9	3	1	4	2	7	3	4	1	7	3
5	3	7	4	5	6	2	1	9	7	2	2
3	7	1	4	4	1	5	3	4	6	1	1

Solution at the end of this article!

Pentomino Product Investigation

Take any pentomino piece and put the numbers 1-5 on it, one in each square. Now multiply together all pairs of squares that

share a side. Add up all your products to find the total for that shape and arrangement of numbers.
e.g.

2	**4**
3	
5	**1**

(2x4) + (2x3) + (3x5) + (5x1) = 34
Now try to find the largest total product for that piece and then the piece and arrangement of numbers that gives the largest product of all.
In carrying out this investigation, the children will come to see the similarities in some of the pieces. They will also have much practice of their tables and mental addition strategies!

Piecing Together the Products
Cut out the pentomino pieces and fit them together to form part of a multiplication square (the rows 4 to 9). Stick it together and then fill in the gaps! Again, a very adaptable activity! It could even include Roman numerals, Chinese characters etc, especially if you used a 10 x 6 rectangle and a number square.

Conclusions

If you have got this far and are now truly hooked on Pentominoes and Katamino, I suggest you look at the following websites, which have even more suggestions for activities and challenges:

www.pentomino.co.uk
www.ex.ac.uk/cimt/puzzles/pentoes/pentmtch.htm
www.pentomino.be.tf/
www.thelighthouseforeducation.co.uk/numeracy/secondary/shapespacemeasure/5polyominoes.ppt

Typing the word 'pentominoes' into one of the search engines will give you even more.

Katamino and pentominoes certainly provide children with a motivational resource to develop their thinking, their problem-solving and their application of mathematical concepts.

Pentominoes provide a prime resource for making the all so important connections between the different areas of mathematics, which help to reinforce and imbed concepts more deeply. Go on, have a go!

'Twenties' Challenge solution

2	3	7	5	8	8	1	6	1	7	5	9
1	5	1	4	1	2	2	6	2	3	4	5
5	9	3	1	4	2	7	3	4	1	7	3
5	3	7	4	5	6	2	1	9	7	2	2
3	7	1	4	4	1	5	3	4	6	1	1